

The degree 9 curve brought to our attention by Mark van Hoeij is given by the equation  $f(u, v) = 0$ , where

$$f(u, v) = v^9 + (504u^2 + 168)v^6 + 405(3u^2 + 1)(u + 1)v^5 - 636(3u^2 + 1)^2v^3 + 324(3u^2 + 1)^2(u + 1)v^2 - \frac{243}{4}(u^2 + 2u + 1)(3u^2 + 1)^2v + 8(3u^2 + 1)^3.$$

The short Weierstrass model for this curve is

$$y^2 = x^3 - 2,$$

and the birational transformation

$$\begin{aligned} u &= \mathcal{U}(x, y), & v &= \mathcal{V}(x, y) \\ x &= \mathcal{X}(u, v), & y &= \mathcal{Y}(u, v) \end{aligned}$$

has the following very large expressions for  $\mathcal{X}(u, v)$  and  $\mathcal{Y}(u, v)$ .

$$\begin{aligned} \mathcal{X}(u, v) &= \frac{1}{324}(9873911006508v^9 + 184025088u^2 - 1417902568944v^6 - 5160721110228u^4v^4 + 9365438847204v^{15}u^3 \\ &+ 1257981164460v^{16}u^3 + 9516423526248v^{14}u^3 - 6890394196620v^6u^4 + 8773414226982u^4v^5 + 85596492898710v^9u^4 \\ &- 1866942310748uv^4 + 6840369584718v^7u^2 + 724997776247v^{10}u + 598459016844v^{15}u^4 - 5132443665762v^9u \\ &+ 88264642041084v^{11}u^2 - 14661363153672v^8u^4 + 57947265663822v^9u^2 + 646387768335v^{10}u^3 + 2291620299286v^{11}u \\ &+ 146419662402684v^{11}u^4 - 12776699434986v^9u^3 - 18833925513438v^{12}u^4 + 12725430897816v^{12}u^3 - 27714249749304v^{13}u^4 \\ &+ 4337428303698v^{11}u^3 - 112568737014135v^{10}u^2 + 9243993447558v^7u^3 + 472897158309v^7u^4 + 393358680v^{19}u \\ &+ 2251833592v^{17}u + 3141735522892v^{14}u - 10251057982720v^{13}u + 405382946084v^{16}u + 3815525531208v^{12}u \\ &+ 8945971242282v^8u^3 - 16093754691732v^8u^2 - 37875240212520v^{13}u^2 - 8603450078088v^{14}u^2 + 9342065762892v^{15}u^2 \\ &+ 438971422680v^{17}u^2 + 252808452v^{18}u^2 + 4083921202800v^{16}u^2 - 454540481280v^{14}u^4 - 29794672081272v^{13}u^3 \\ &+ 11602358058006v^{12}u^2 + 1712677819824u^3v^3 - 2111267400504u^2v^4 + 467468765808uv^3 + 3162029532v^{18}u \\ &+ 3133154180532v^{15}u - 173092554483939v^{10}u^4 + 2777206977239uv^8 - 1564766849628u^5v^4 + 930814567200u^5v^3 \\ &- 6609409983210uv^6 - 6122415882120u^3v^4 + 3745857901465uv^7 - 272965632v - 449839104u \\ &- 18987657164712u^3v^6 - 6544853332308v^6u^2 + 14199336583836v^5u^3 + 4629222509412v^5u^2 + 4638832948658v^5u \\ &+ 1684236336984v^3u^4 + 605498729184v^3u^2 - 306268130304v^2u^5 - 290222754048v^2u^4 - 327807697920v^2u^3 \\ &- 96403266048v^2u^2 - 75239440384v^2u + 19436544000v^4u + 5659951104v^2u + 54410969088v^5u^5 \\ &+ 41708470272vu^3 + 7857160192vu + 568386468694v^5 + 14695538952v^3 + 112550656v^2 \\ &- 2699034624u^3 + 276037632u^4 - 4048551936u^5 - 18369327072670v^{10} + 2238650790029v^7 \\ &- 130342343476v^4 - 3771348892964v^8 + 13202176151552v^{11} + 5949462213870v^{12} - 9551837680664v^{13} \\ &- 2811787045912v^{14} + 3041689935732v^{15} + 1378870044328v^{16} + 123807236624v^{17} + 3661017000v^{18} \\ &+ 6004565104v^{19} + 869576472v^{20} - 4938595250814v^{10}u^5 + 849462092946v^5u^5 + 2508813567810v^6u^5 \\ &+ 8257206117276v^9u^5 + 4061645593506v^{12}u^5 - 7568285995512v^{11}u^5 + 1616359962303v^8u^5 - 5907060886815v^7u^5 \\ &+ 323268828714v^{14}u^5 + 2657183628636v^{13}u^5 + 30670848)/(u^5v^5(14414517v^9 + 75504636v^8 \\ &- 101364176v^7 - 379377892v^6 + 1037132388v^5 - 1104918944v^4 + 637336208v^3 \\ &- 209885328v^2 + 37233664v - 2768896)), \end{aligned}$$

and

$$\begin{aligned}
\mathcal{Y}(u, v) = & -\frac{1}{324}(-4623102566081160 v^9 + 742970621952 u^2 + 2987589120133356 v^6 + 803578994483508 u^4 v^4 \\
& + 7996478383473120 v^{15} u^3 + 11075040251332884 v^{16} u^3 - 39391285129024464 v^{14} u^3 - 3397684252906404 v^6 u^4 \\
& + 496586999057184 u^4 v^5 - 73380715275443544 v^9 u^4 + 733777044399924 u v^4 - 7601619139073076 v^7 u^2 \\
& + 4951743290817744 v^{10} u - 1801337300387712 v^{15} u^4 + 870998257426338 v^9 u - 179477330345389593 v^{11} u^2 \\
& + 15328240524619461 v^8 u^4 - 38306878193147184 v^9 u^2 + 28180599350765472 v^{10} u^3 - 4927608512877099 v^{11} u \\
& - 268934969242460421 v^{11} u^4 - 5502519220061772 v^9 u^3 + 183404796346709100 v^{12} u^4 - 6612564308537106 v^{12} u^3 \\
& - 9544170648361500 v^{13} u^4 - 22354788801193587 v^{11} u^3 + 116989037960220120 v^{10} u^2 + 10045474060933908 v^7 u^3 \\
& + 2328922246538124 v^7 u^4 + 123828436992 + 2908950456276 v^{19} u + 507111033549336 v^{17} u \\
& - 13398993480085392 v^{14} u + 10573119726916368 v^{13} u + 3701441431147188 v^{16} u - 1434841711812774 v^{12} u \\
& - 12848068842067782 v^8 u^3 + 9093401049080394 v^8 u^2 + 30674810211471372 v^{13} u^2 - 46564027365875184 v^{14} u^2 \\
& - 14621314936108824 v^{15} u^2 + 5115021999765984 v^{17} u^2 + 557093376329184 v^{18} u^2 + 10768167113970108 v^{16} u^2 \\
& - 34973939870503920 v^{14} u^4 + 33964701033157200 v^{13} u^3 + 108774332909298468 v^{12} u^2 - 701534479965696 u^3 v^3 \\
& + 2190288203886696 u^2 v^4 - 233051738904576 u v^3 + 1629516295296 v^{18} u + 2632765913351676 v^{15} u \\
& + 185767466545143720 v^{10} u^4 - 4873225990756227 u v^8 - 255263669671356 u^5 v^4 - 7137789755904 u^5 v^3 \\
& - 22889881383300 u v^6 + 2116243243309320 u^3 v^4 + 3221152600930044 u v^7 + 217883130677550 v^{15} u^5 \\
& + 181183825836 v^{19} u^2 + 1551729104995752 v^{17} u^3 + 421678356840 v^{20} u + 604855898704044 v^{16} u^4 \\
& - 2494774378496 v + 96674512896 u - 544526022396456 u^3 v^6 + 7830003138860088 v^6 u^2 \\
& - 3121125471050112 v^5 u^3 - 5072925134545200 v^5 u^2 - 1130598866546520 v^5 u - 545679703832832 v^3 u^4 \\
& - 646798417256448 v^3 u^2 + 32666896982016 v^2 u^5 + 155084034834432 v^2 u^4 + 131395948019712 v^2 u^3 \\
& + 126125255835648 v^2 u^2 + 40168994119680 v^2 u - 21215768739840 v u^4 - 14556246048768 v u^2 \\
& - 9314769567744 v u^5 - 13320023703552 v u^3 - 3405033504768 v u - 1746154813211952 v^5 \\
& - 154968394215168 v^3 + 24810192519168 v^2 + 580047077376 u^3 + 1114455932928 u^4 + 870070616064 u^5 \\
& + 18368607723945760 v^{10} - 2793209049000168 v^7 + 640809513019620 v^4 + 1330649562825489 v^8 \\
& - 29956232771134374 v^{11} + 15881166812990736 v^{12} + 11294505184532796 v^{13} - 11642690335627920 v^{14} \\
& - 4673589255256248 v^{15} + 3514365068226756 v^{16} + 1735366814398608 v^{17} + 154492517897472 v^{18} \\
& + 2930091819400 v^{19} + 7487662063536 v^{20} + 1102403399472 v^{21} + 40099920708787632 v^{10} u^5 \\
& + 811864325447064 v^5 u^5 - 1425253176352980 v^6 u^5 - 24460097213718054 v^9 u^5 - 6989617706062536 v^{12} u^5 \\
& - 22748353324821054 v^{11} u^5 + 5369707693979433 v^8 u^5 + 1130785587568224 v^7 u^5 + 2469224080620576 v^{14} u^5 \\
& + 6772575278871252 v^{13} u^5) / (u^5 v^5 (1167575877 v^{10} + 5654610972 v^9 - 10626646608 v^8 - 27485955620 v^7 \\
& + 96147815972 v^6 - 122686670880 v^5 + 86981639056 v^4 - 37395470224 v^3 \\
& + 9732257280 v^2 - 1415757824 v + 88604672)).
\end{aligned}$$